BOWLAND MATHS
Professional development

Questioning and reasoning

'How do I get my pupils to think, reason and explain?'

Module overview

This module considers:

- characteristics of questioning that encourages pupils to listen, think and reason;
- ways in which you might encourage pupils to provide extended, thoughtful answers, without being afraid of making mistakes;
- the value of modelling reasoning by 'thinking aloud' with your class.

This guide is intended for use alongside the *Bowland Maths DVD* or website, which include a short introductory video for each of the activities, longer videos of lessons and teacher discussions and links to all the handouts and ICT-based problems.

Introductory session — 1 hour

- Think about why we ask questions.
- Compare 'effective' and 'ineffective' questioning.
- Observe a lesson
- Plan a lesson for effective questioning

Into the classroom — 1 hour

- Introduce the problem and give time for individuals to think
- Collect initial ideas on the board
- Pupils work on the problem
- Whole class discusses the approaches being used
- Pupils have a second go at the problem.
- Whole class reports on their reasoning

Follow-up session — 1 hour

- Report and reflect on the lesson
- Solve a problem, "thinking aloud"
- Watch some teachers "thinking aloud"
- Plan questioning and "thinking aloud" for future lessons

Resources Needed

Handout 1	Thinking about why we ask questions
Handout 2	Comparing 'effective' and 'ineffective' questioning
Handout 3	Characteristics of effective questioning
Handout 4	Problems to try
Software	Multiplication grids (optional)
Handout 5	Planning for effective questioning
Handout 6	Comments on the problems
Handout 7	Suggested further reading

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Introductory session

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Introduction — The importance of questioning

Question and answer is perhaps the most common form of interaction between teachers and pupils. Teachers use questions for many reasons: to gain pupils' attention, to monitor their understanding or to promote thinking and reasoning.

Pupils do not always appreciate the intentions behind the teacher's questions. This is seen for example, when they try to guess 'the right answer' rather than give a thoughtful explanation.

In this module, we therefore use tasks that don't have a single 'right answer' and look closely at how careful questioning can be used to promote thinking, reasoning and explaining.

Activity 1 — Think about why we ask questions — 5 minutes

There are many different types of questions and they serve many different purposes.

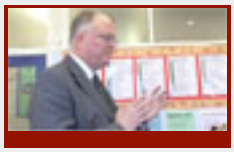
- What different types of questions are there?
- What different purposes do your questions serve?
- Which types do you use most frequently?

List your ideas on Handout 1.

There are, of course, many possible reasons for asking questions, including the following:

- to interest, engage and challenge;
- to assess prior knowledge and understanding;
- to stimulate recall, mobilise existing knowledge and experience in order to create new understanding and meaning;
- to focus thinking on key concepts and issues;
- to help learners extend their thinking from the concrete and factual to the analytical and evaluative;
- to lead learners through a planned sequence which progressively establishes key understandings;
- to promote reasoning, problem solving, evaluation and the formation of hypotheses;
- to promote learners' thinking about the way they have learned;
- to help pupils see connections between different parts of mathematics or with different contexts.

Activity 2 **Compare 'effective' and 'ineffective' questioning** **15 minutes**



Record your ideas on the following questions on Handout 2.

- What are the common mistakes *you* tend to make when asking questions?
- What are their effects?
- What types of questions encourage thinking and reasoning?
- Give some examples that you have recently used.

Now look at Handout 3. This describes some characteristics of effective questioning.

- Reflect on the implications of these ideas for your own practice.

The following is a list of some of the more common mistakes that teachers make:

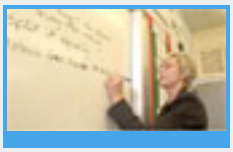
- Asking too many trivial or irrelevant questions.
- Asking a question and answering it yourself.
- Simplifying the question when pupils don't immediately respond.
- Asking questions of only the brightest or most likeable pupils.
- Asking several questions at once.
- Only asking closed questions that allow one possible answer.
- Asking 'guess what is in my head' questions, where you know the answer you want to hear and you ignore or reject answers that are different.
- Judging every pupil response with 'well done', 'nearly there' 'not quite'. ('Well done' can discourage alternative ideas being offered)
- Not giving pupils time to think or discuss before responding.
- Ignoring incorrect answers and moving on.
- Using simple questions as a way to make explanation seem interactive.
- Asking in a manner that makes pupils afraid to respond.

Research shows that effective questioning displays the following five characteristics:

- The teacher plans questions that encourage thinking and reasoning.
- Everyone is included.
- Pupils are given time to think.
- The teacher avoids judging pupils' responses.
- Pupils' responses are followed up in ways that encourage deeper thinking.

Handout 3 elaborates each of these points.

Activity 3 — Observe a lesson — 20 minutes



Have a go at the *Sharing petrol costs* problem on Handout 4.

Now watch Gwen's lesson, which uses this problem.

As you watch the video clip consider the following questions:

- How does Gwen use questions?
- How does she respond to the wide range of answers?
- What is she trying to achieve with her interactions?

Later, you may wish to come back and watch Jeff's lesson using *Aircraft turn-around time* or Chris's lesson using the *Multiplication grids* software.

Activity 4 — Plan a lesson for effective questioning — 20 minutes

Choose one of the problems from Handout 4 to try with your class.

Use the prompts on Handout 5 to plan a lesson that will promote thinking and reasoning.

- How will you organise the classroom and the resources?
- How will you introduce the questioning session?
- Which ground rules will you establish?
- What will be your first question?
- How will you give time for pupils to think before responding?
- Will you need to intervene at some point to refocus or discuss different strategies they are using?
- What questions will you use in plenary discussions during or towards the end of the lesson?

Because you are focusing on the questions that you use and the way that the pupils answer those questions we suggest that you audio-record some whole class questioning in your lesson for discussion in the follow-up session

In a group professional development session, it is helpful if each participant chooses the same problem, as this will enable you to have a good discussion afterwards.

There are comments on each of the problems on Handout 6.

This is the end of the *Introductory session*. After you have tried out your lesson with your own pupils, return for the *Follow-up session*.

Resources to support the lessons, and suggested lesson plans, can be found in the *Into the classroom* session.

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Into the classroom

The following suggestions describe one possible approach to using the problems with pupils. This may take one or two lessons, depending on the class.

Introduce the problem, and give time for individuals to think — 5 minutes

Issue each pupil with just one of the three problems.
Explain that in this lesson you are expecting everyone to think things through and to contribute:

> *Today I am going to make sure you have plenty of time to think. I will give you a problem and I want you to think about how to get started with it for a few minutes. I will then ask for your ideas. There is more than one good way of doing this!*
>
> *No hands up, I will tell you when I want answers and who is to answer.*
> *Now here's the question I want you to think about …*

Explain how pupils are expected to start work on the problem:

> **Sharing petrol costs**.
> *Read through the problem. How can we get started on this problem? What is known and what is unknown? What assumptions should we make? Remember I don't want answers yet; I want to know your ideas for getting started. You have exactly 1 minute to think starting now!*
>
> **Multiplication Grids**.
> *Read through and think about the problem on your own for a couple of minutes. Then I'll ask you to share ideas in pairs or threes. Then I am going to ask the question "Have you any ideas for getting started?"*
>
> **Aircraft turn-round time.**
> *Look at all the jobs that need doing before a plane is turned round and ready to fly again. You have a minute to discuss with the person sitting next to you how you might go about finding the shortest turn round time – starting now.*

Collect initial ideas on the board — 5 minutes

After the 'thinking time', pose the problem again then use the alternatives to questioning to generate discussion. (Record this part for later discussion if possible).

> *Right let's get started, what did you think about, Jav?*
> *Thanks for that explanation, Jav.*
> *Does anyone have any comments on Jav's ideas?*
> *Yes I can see that, what else might we think about Sarah?*
> *We've talked about three good ideas so far; does anyone have anything really different?*

Note that these questions are general and strategic. Do not comment on the specifics of the responses at this stage, even if pupils press you to tell them what is 'best' or who is 'right'. Instead, simply record these ideas on the board, or get the pupils to do this as they explain. That way the ideas will be there for the class to consider as they start to solve the problem. Remind them that although they have heard several strategies that will help them get started, that they should choose just one of them to start with.

Explain what pupils should do when they are stuck:

> *If you get stuck, think about the ways of tackling the problem we have talked about. Maybe you could try another one? Remember this lesson is about thinking and reasoning things out, so sit quietly and think about what you could do, then you could talk to a friend about what you are thinking. You are on your own, get going!*

Now set a target, reminding them to think about the reasons they make decisions as they work:

> *Right, now I'm giving you twenty minutes to work on the problem by yourselves. Then I'm going to ask you some questions about what you have done and why you think the ideas you tried worked or didn't work.*

Pupils work on the problem — 20 minutes

Allow pupils time to engage with the problems. When they ask questions, ask them a question that offers strategic guidance rather than technical help. Use Handout 3 for ideas.

For example:

> *Which way did you decide to use to start? Why?*
> *What have you found out? How did you do that?*
> *What didn't work? Why? What might work?*
>
> *Think things out for yourself or between you – only call in the professional when you have tried everything else.*

Whole class discusses the approaches being used — 10 minutes

When most pupils have made significant progress with the problem, ask the pupils about the way that they are working. (It may be helpful to record this part for later discussion).

> *We are going to review progress so far.*
> *I don't want answers I want strategies and ideas.*
> *I want to know what you have done so far. What have you tried that didn't work? Why didn't it work?*
> *What have you tried that seems to be successful?*
> (5 sec pause for thinking)
> *Right let's start with the first question – what did you try that didn't work and why?*

When exploring the unsuccessful ideas remember to ask "What was the unhelpful idea here? What would have made it work?" You are making sure that the pupils know its fine to make mistakes and take wrong turns when solving problems but it's the successful ideas that you want, so after a few minutes ask for them.

Sharing petrol costs:
What assumptions made the petrol money sharing much easier? Can you justify your ideas?

Multiplication Grids:
What ideas are helping you to put the right numbers in the right places on the grid?

Aircraft turn-round time:
What representations helped you to organise your approach?

The idea is to provide models that will help pupils to make more progress on the problem. Make sure that the pupils listen to the ideas given. Ask the next pupil to comment on how similar or different their idea is to those offered previously, rather than take isolated answers.

Pupils have a second go at the problem 10 minutes

Encourage pupils to return to the problem and continue working on it using some of the ideas that have been shared.

Whole class reports on their reasoning 10 minutes

Ask pupils to take turns at presenting their reasoning to the class.

What ideas did you have that worked? Tell us why they worked.

Focus on the thinking rather than the answers. Make sure they know there is no one right answer to these problems. Ask questions such as:

What was it about Sam's ideas that enabled her to solve the problem easily?

Why might you not use the same solution as Sam?

What did Josh do that was particularly inventive or different?

What ideas did Nils have that you could use?

If you want to set homework, the pupils should now have a series of ideas that would help them both finish of the problem and explain how they did it. The lesson was about thinking and reasoning so asking the pupils to explain what they tried and why is more important than just achieving an answer.

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Follow-up session

Activity 1 — Report and reflect on the lesson — 20 minutes

Reflect on the lesson you have just taught and listen to any recordings you made of your questioning.

- What strategies did you use?
- Which questions appeared to promote thoughtful and reasoned responses from pupils? Why was this?
- Which questions didn't work so well? Why was this?

You may like to watch the video of the three teachers discussing these same issues.

Activity 2 — Solve a problem, "thinking aloud" — 10 minutes

Problem solving is an invisible, messy process that goes on inside people's heads. One reason why some pupils are reluctant to persist in problem solving is that they do not recognise that it is perfectly natural to get stuck, make mistakes, backtrack, look for alternative strategies and so on.

It is helpful for a teacher to model this process by tackling a problem from start to finish, thinking aloud and involving the class by careful questioning.

Try working out an answer to the following problem, thinking aloud as you do so:

About how many dentists are there in the UK?

Afterwards think what it would feel like, doing this with a class, not knowing the answer beforehand.

If you are working with colleagues, one could take the role of the teacher and try to tackle the problem step by step, thinking aloud. The others could take the role of pupils and try to assist when asked to do so.

- What should we try first?
- What helpful representation can we use here?
- How can we check this step?
- Where have we gone wrong here?
- Is this heading in the right direction?

Activity 3 — Watch some teachers "thinking aloud" — 15 minutes

Watch Gwen, Jeff and Chris as they solve the dentists problem together.

After watching the video clip, compare their comments with your own experience:

- Did you feel under similar pressure?
- Would you feel afraid of thinking aloud like this in front of a class? Why?
- What other ideas have you got for making thinking more "visible"?

The following strategies illustrate the last point.

Speaking the teacher's mind…
Before solving a problem, appoint one or two pupils to stand at the front and 'speak your mind'. They try to give reasons for each step. (E.g. Why did I do put the 7 here on this multiplication grid?).

Evaluating other pupils' reasoning...
Photocopy a few anonymous pieces of pupils' work (or create your own exemplars) and ask the class to work in groups of two or three to analyse and discuss where good reasoning is shown and to give suggestions of where the reasoning could be clarified or improved. Alternatively, write out solutions, incorporating errors, misconceptions and inconsistencies and ask the pupils to work together to find, correct and comment on the 'errors in reasoning'. They should also comment on where the reasoning was good so that they can use these ideas again.

Activity 4 — Plan questioning and "thinking aloud" for future lessons — 15 minutes

Plan some ways of applying what you have learned in this PD module to other mathematics lessons that you teach.

Choose one topic that you plan to teach soon.

Think of some ways in which you might build in effective questioning and "thinking aloud" into your teaching of that topic.

Further Reading

See Handout 7 for suggested further reading.

1 Thinking about why we ask questions

What different types of questions are there?

What different purposes do your questions serve?

Which types of questions do you use most frequently?

2 Comparing 'effective' and 'ineffective' questioning

What common mistakes do you make when asking questions?

List the common mistakes that you make when asking questions.
What are the unintended effects of each of these mistakes?

Common mistake	Unintended effect

What types of questions seem to encourage thinking and reasoning?

Give a few examples that you have recently used.

3 Characteristics of effective questioning

(i) Plan to use questions that encourage thinking and reasoning

Really effective questions are planned beforehand. It is helpful to plan *sequences* of questions that build on and extend pupils' thinking. A good questioner, of course, remains flexible and allows time to follow up responses. The following suggested questions are related to the key processes in the Programmes of Study:

Representing	• How can we get started on this problem? • What maths might be useful here? • What sort of diagram might be helpful? • Can we invent a simple notation for this? • How can we simplify this problem? • What is known and what is unknown? • What assumptions might we make?
Reasoning	• Where have we seen something like this before? • What is fixed here, and what can we change? • What is the same and what is different here? • What would happen if I changed this.. to this...? • Is this approach going anywhere? • What will you do when you get that answer? • This is just a special case of ... what? • Can you form any hypotheses? • Can you think of any counterexamples? • What mistakes have we made? • Can you suggest a different way of doing this? • What conclusions can you make from this data? • How can we check this calculation without doing it all again? • What is a sensible way to record this?
Interpreting and evaluating	• How can you best display your data? • Is it better to use this type of chart or that one? Why? • What patterns can you see in this data? • What reasons might there be for these patterns? • Can you give me a convincing argument for that statement? • Do you think that answer is reasonable? Why? • How can you be 100% sure that is true? Convince me! • What do you think of Anne's argument? Why? • Which method might be best to use here? Why?
Communicate and reflect	• What method did you use? • What other methods have you considered? • Which of your methods was the best? Why? • Which method was the quickest? • Where have you seen a problem like this before? • What methods did you use last time? Would they have worked here? • What helpful strategies have we learned for next time?

(ii) Ask questions in ways that include everyone

It is very important that everyone is included in thinking about the questions asked. Here are three ways that teachers have tried to achieve this:

- **Use a 'no hands up' rule**. After a few hands have gone up some pupils stop thinking because they know that the teacher will not ask them. When pupils have their hands up they too stop thinking as they already have the answer they want. "No hands up" encourages everyone to keep thinking as anyone may be called upon to respond.

- **Ask questions that encourage a range of responses.** Rather than asking for specific right answers, ask for ideas and suggestions: "How can we get started on this?", "What do you notice about this?" Everyone will then be able to offer a response.

- **Avoid teacher - student - teacher - student 'ping pong'.** Encourage pupils to listen to and to reply to each other's responses. Aim for a pattern more like: teacher - pupil A - pupil B - pupil C - teacher.

- **Arrange the room to encourage participation.** Think about where pupils are sitting – are there some who cannot hear? Can pupils see and hear one another so that they can respond to the points that another pupil makes? It is often better to sit pupils in a U-shape, if possible.

(iii) Give pupils time to think

The time interval between a teacher asking a question and supplying the answer herself, or following up with an additional question or comment, is commonly called 'wait time'. For many teachers, the mean wait time is less than one second (Rowe (1974)[1]). When teachers increase this wait time to between three and five seconds the research shows that pupils begin to:

- respond at greater length and with greater confidence;
- offer more unsolicited, but appropriate, responses;
- offer more diverse, alternative explanations;
- relate responses to those from other pupils.

Increasing wait time is difficult. Silence in a classroom can be hard to bear!

- **Talk to students about 'wait time'**. Make sure that pupils *know* that they must take time to think before responding. (Some teachers even make themselves wait by counting slowly to themselves: "One, two, three, four, got to wait a little more"!)

- **Use "Think - Pair - Share".** Ask the question, give 10 seconds thinking time and then allow 30 seconds for talking to a partner. After this, everyone should be ready with an answer and they should know that anyone may be asked for what they think.

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[1] Rowe, M. B. 1974. *Wait time and rewards as instructional variables, their influence on language, logic and fate control*. Journal of Research in Science Teaching 11:81-94.

- **Use mini whiteboards** - Ask the pupils to spend 30 seconds thinking about the problem and jotting ideas for the solution onto their mini whiteboards. Then ask the pupils to share the ideas they had for starting the problem

(iv) Avoid judging pupils' responses

Interestingly, Rowe (1974) found that if a teacher made judgmental comments, even positive ones such as “Well done!”, then this negatively affected pupils' verbal performance even with the lengthened wait times. Task persistence was greatest where verbal rewards were fewer.

When a teacher judges every response with 'yes', 'good' , 'nearly' and so on, pupils are likely to reason to themselves:

> *"The teacher said that was good. That is not what I was going to say. So what I was going to say cannot be good. So I won't say anything."*

Ask open questions that permit a greater variety of responses and reply to pupils with comments that do not close off alternative ideas.

> *"Thank you for that, that is really interesting. What other ideas do people have?"*

(v) Follow up pupils' responses in ways that encourage deeper thinking

The following approaches encourage further thinking and dialogue:

Ask pupils to repeat their explanation	• Can you just say that again?
Invite pupils to elaborate	• Can you just say a little more about that …
Challenge pupils to offer a reason	• Can you explain why that works?
Cue alternative responses	• Can you suggest another way of doing this?
Support with non-verbal interest	• Nod head, rotate hand to indicate that you want more ...
Encourage pupils to speculate.	• What would happen if ...?
Make challenging statements	• Someone in this group said … were they right?
Allow rehearsal of responses	• Try out the answer on your partner first.
Encourage pupils to ask questions	• Would anyone like to ask Pat a question about that?
Ask pupils to think aloud	• Can you go through that step by step?
Encourage pupils to make connections	• Can you remember something else we did like this ...?
Thinking aloud with pupils	• Let's think this through together ...

4 Problems to try

Modelling and explaining: Sharing petrol costs

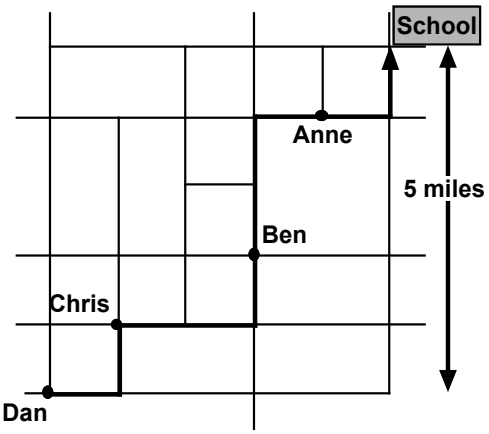
Each day Dan's mum drives him to school.

On the way, she picks up 3 of Dan's friends, Chris, Ben and Anne.

Each afternoon, she returns by the same route and drops them off at their homes.

At the end of a term, the four students decide to pay a sum of £100 towards the cost of petrol.

How should they share out the cost?
Find some reasonable solutions and say which you think is best and why.

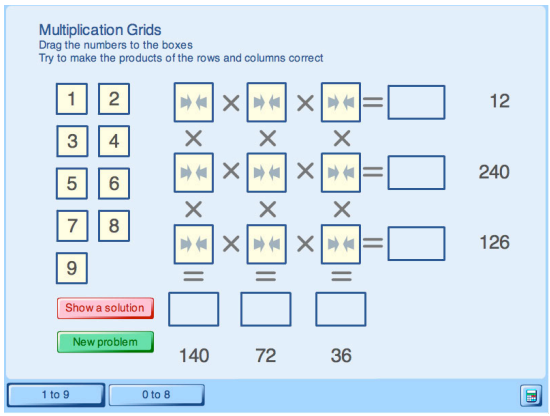


This map shows where each person lives and the route taken.

Solving logic puzzles: Multiplication grids

Drag the numbers 1 to 9 into the yellow spaces to make the products of the rows and columns correct.
There is more than one puzzle to solve!

What advice would you give someone else to help them solve puzzles like this?



Planning and organising: Aircraft turn-round time

Between landing and taking off, the following jobs need to be done on an aircraft.

	Job	Time needed
A	Get passengers out of the cabin and off the plane	10 minutes
B	Clean the cabin	20 minutes
C	Refuel the plane	40 minutes
D	Unload the baggage from the cargo hold beneath the plane	25 minutes
E	Get new passengers on the plane	25 minutes
F	Load the new baggage into the cargo hold	35 minutes
G	Do a final safety check before take-off	5 minutes

What is the shortest time needed to do all these jobs?
Would it make any difference to this time if the people could get off more quickly (from the front and rear of the plane)?

5 Planning for effective questioning

Plan how you will arrange the room and the resources needed	Arrange pupils so that they can see and hear one another as well as the teacher. You may need to rearrange chairs in a U shape or the pupils could move and 'perch' closer together. Or maybe you will move to the back of the room so that the question is the focus of attention and not the teacher.
Plan how you will introduce the questioning session	Silence will be hard for you to bear in the classroom but the pupils may find it confusing or even threatening. Explain why there will be times of quiet. For example:
Plan how you will establish the ground rules	If you are using 'No hands up' then you will need to explain this to the pupils. Some teachers have had to ask their pupils to sit on their hands so that they remember not to put their hands up. The pupils will be allowed to put their hands up to ask a question, so if a hand shoots up remember to ask them what question they would like to ask. The pupils may also be used to giving short answers so you could introduce a minimum length rule e.g. 'your answer must be five words in length as a minimum'.
Plan the first question that you will use	Plan the first question and think about how you will continue. You cannot plan this exactly as it will depend on the answers that the pupils give but you might, for example, plan ▪ to take one answer and then ask others what they think about the reasoning given ▪ to take two or three answers without comment then ask the next person to say what is similar or different about those answers
Plan how you will give thinking time	▪ Will you allow 3-5 seconds between asking a question and expecting an answer? ▪ Will you ask the pupils to think – pair – share, giving 30 seconds for talking to a partner before offering an idea in whole class discussion? ▪ Will you use another strategy that allows the pupils time to think?
Plan how and when you will intervene	Will you need to intervene at some point to refocus pupils' attention or discuss different strategies they are using? Have one or two questions ready to ask part way through the lesson to check on their progress and their learning.
Plan what questions you could use for the plenary at the end of the lesson	Try not to pass judgments on their responses while they do this or this may influence subsequent contributions.

6 Comments on the problems

Modelling and explaining: Sharing petrol costs

Links to Case Studies

In the Case Studies, pupils are sometimes asked to create a method for sharing a resource. For example, in **Water availability**, pupils are asked to find a fair way to distribute water among different countries. This invites them to create a mathematical method (or model) and discuss its validity. In this short 'sharing petrol' task, pupils are asked to find alternative models and decide which is best. This can be difficult as there is often a tendency to see 'lock onto' one method very quickly and resist the invitation to search for alternatives.

Sample solution

Two reasoned methods are shown below. Which do you consider better?

Method 1:

This is to share the cost in the proportion to the road distance people live from school: 2: 5: 8: 10. So:

Anne pays	£8
Ben pays	£20
Chris pays	£32
Dan pays	£40

Method 2:

Assume that, altogether, people will need to pay £10 per mile. Costs are shared out as follows:

	Anne	**Ben**	**Chris**	**Dan**
Last 2 miles £20	£5	£5	£5	£5
Next 3 miles £30		£10	£10	£10
Next 3 miles £30			£15	£15
First 2 miles £20				£20

Anne pays	£5
Ben pays	£15
Chris pays	£30
Dan pays	£50

Solving logic puzzles: Multiplication grids

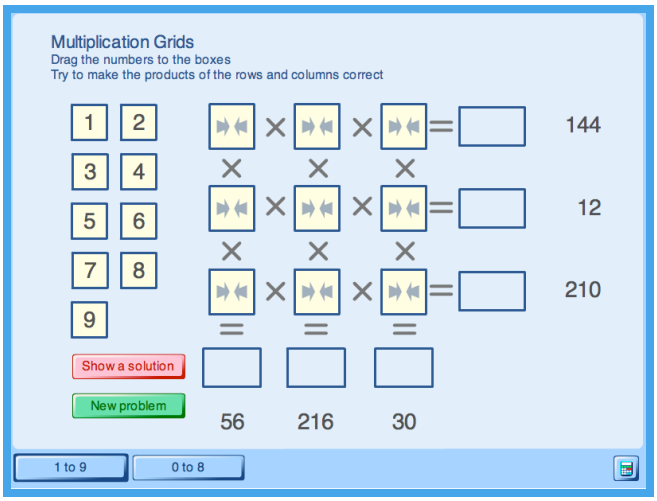
Links to Case Studies

There are a number of Case Studies that contain logic problems that are quite abstract in nature. These are often situated in the context of computer games. The essence here is to proceed logically and systematically.

Sample solution

Two versions of the Multiplication Grid puzzle are included, one that is based on placing the numbers 0 to 8 and one that involves placing 1 to 9. As the puzzles are generated randomly, pupils will need to develop general strategies. They may discover that the zero (when used) and the primes 5 and 7 are the easiest to place. When these are placed, other numbers may often be deduced. Sometimes there is more than one correct answer!

For example, we must place 1 to 9 in the puzzle shown below.



- 5 must go in the third row and the third column, because only 30 and 210 are divisible by 5.
- 7 must go into the third row and first column (only 56 and 210 are divisible by 7)
- 6 must now go in between (to make the product 210).
- Now 216÷6=36. So the middle column must contain 9 and 4. Since 12 is not divisible by 9, 9 must go into the centre of the first row and 4 must go into the central box.
- We are now left with only 1,2,3,8. The only two remaining products divisible by 8 are 144 and 56, so 8 must go in the first row, first column. This forces 1 into the second row, first column. The remaining two numbers are now easy to place. This results in the unique solution (in this case):

8	*9*	*2*
1	*4*	*3*
7	6	5

Planning and organising: Aircraft turn-round time

Links to Case Studies

There are a number of Case Studies that require planning in order to find an optimal solution. Examples are:

- **Outbreak**
- **Product wars**
- **Mystery tours**
- **Highway link design**

Sample solution

Pupils are likely to begin this problem by simply adding all the times together. Some may then notice that some jobs may be done simultaneously.

This kind of problem is more easily solved if a helpful representation is found. Mathematicians will recognise this is as a critical path analysis problem and use networks. Others may prefer to use a table or other representation (for example, by having each job on a different scrap of paper and arranging these). A suitable table is shown below:

<table>
<tr><th>Time completed</th><th>Jobs being done inside the plane</th><th>Jobs being done outside the plane</th><th>Jobs being done in the cargo hold</th></tr>
<tr><td>0</td><td rowspan="3">A: Get passengers out of the cabin and off the plane</td><td rowspan="8">C: Refuel the plane</td><td rowspan="4">D: Unload the baggage</td></tr>
<tr><td>5</td></tr>
<tr><td>10</td></tr>
<tr><td>15</td><td rowspan="4">B: Clean the cabin</td></tr>
<tr><td>20</td></tr>
<tr><td>25</td></tr>
<tr><td>30</td><td rowspan="6">F: Load the new baggage</td></tr>
<tr><td>35</td><td rowspan="5">E: Get new passengers on the plane</td></tr>
<tr><td>40</td></tr>
<tr><td>45</td></tr>
<tr><td>50</td></tr>
<tr><td>55</td><td rowspan="2">Finished</td></tr>
<tr><td>60</td><td>Wait</td></tr>
<tr><td>65</td><td>F: Final safety check</td><td>Finished</td></tr>
</table>

This table shows that the plane will be ready in 65 minutes. Even if the people could get off in a shorter time, it would not speed things up as the final safety checks couldn't take place until after all the baggage had been loaded.

7 Suggested further reading

Effective collection of questions for mathematical thinking
Bills, C., Bills, L., Watson A., J. Mason (2004), *Thinkers*, Association of Teachers of Mathematics, Derby. www.atm.org.uk

More effective questions for promoting mathematical thinking
Bills, L. Latham, P. and Williams, H. (2002) 'Encouraging all learners to think' *Mathematics Teaching*, 181, pp 14-16
http://www.atm.org.uk/mt/archive/mt181files/ATM-MT181-14-16.pdf

Questioning to enable effective learning and assessment for learning
Lee, C. (2006) Language for Learning Mathematics – assessment for learning in practice. Open University Press.

Questioning in the mathematics classroom, what really happens and what could happen?
Martin, N. (2003), 'Questioning styles', *Mathematics Teaching*, 184, pp 18-19
http://www.atm.org.uk/mt/archive/mt184files/ATM-MT184-18-19-mo.pdf

Is questioning really important?
Smith, J. (1986), 'Questioning Questioning', *Mathematics Teaching,* 115, p47.

The questions that make pupils think mathematically
Watson, A. and Mason, J. (1998) *Questions and Prompts for Mathematical Thinking,* Association of Teachers of Mathematics Derby, www.atm.org.uk