BOWLAND
MATHS
Professional development

Involving pupils in self- and peer-assessment

How can pupils help each other to use the Key Processes?

Module overview

"... self-assessment by pupils, far from being a luxury, is in fact an essential component of formative assessment. Where anyone is trying to learn, feedback about their efforts has three elements—the desired goal, the evidence about their present position, and some understanding of a way to close the gap between the two. All three must to a degree be understood by anyone before they can take action to improve their learning" (Black & Wiliam, 1998).

This is particularly true when the focus of the assessment is on Key Processes. Many pupils do not understand the nature and importance of these processes in mathematics. If a pupil's goal is only to get 'the right answer', then she will not attend to the deeper purposes of the lesson.

This module encourages discussion of the following issues:

- How can we help pupils to become aware of the Key Processes, and their importance in problem solving?
- How can we encourage pupils to take more responsibility for their own learning of Key Processes?
- How can pupils be encouraged to assess and improve each other's work?

Introductory session **1 hour**

- Explore how pupils may become aware of Key Processes
- Discuss how pupils can learn from sample responses
- Discuss how pupils can assess their own work
- Consider how pupils can collaboratively improve work
- Plan to use peer and self-assessment strategies

Into the classroom **1 hour**

- Pupils work on the problem on their own
- Teacher assesses their work
- Pupils work together improving their work or sample work
- Pupils receive further help from other groups and from provided 'progression steps'
- Plenary discussion on approaches used and changes made

Follow-up session **1 hour**

- Report and reflect on the assessment lesson
- Discuss strategies for differentiation
- Discuss ways of helping pupils that struggle
- Discuss ways of stretching pupils that succeed
- Plan assessment strategies for future lessons

Resources needed

Handout 1: Strategies for helping pupils to become more aware of Key Processes
Handout 2: Three assessment tasks and five sample responses on each
Handout 3: Meeting the needs of all pupils
Handout 4: Meeting the needs of all pupils – some comments to consider
Handout 5: Suggestions for further reading

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Introductory session

Involving pupils in self- and peer- assessment

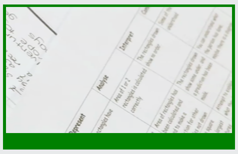
How can pupils help each other to use the Key Processes?

Introduction — Involving pupils in self- and peer- assessment

This module explores how students can assess and develop their own abilities to use the Key Processes when problem solving. Self and peer assessment have the potential to help pupils become more aware of the goals of their learning and of the ways in which they can improve their own work to achieve these goals. As this awareness grows, pupils become more autonomous learners.

In this module, we follow three teachers: Sheena, Emma and Shane, from Arthur Terry School in Sutton Coldfield, as they explore different ways of helping pupils to assess and improve their own work.

Activity 1 — Explore how pupils may become aware of Key Processes — 10 minutes

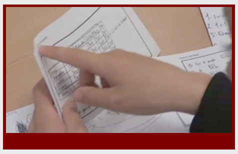


Handout 1 presents a number of suggestions for making pupils more aware of their own progress in understanding and working on Key Processes.

- Discuss the advantages and disadvantages of each suggestion
- Can you think of any other ways of making pupils more aware of the objectives of these tasks?
- Can you think of any other ways of engaging pupils in peer assessment?

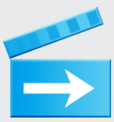
This module considers some of these suggestions in more depth.

Activity 2 — Consider how pupils can learn from sample responses — 20 minutes



In an earlier lesson, Sheena's pupils worked individually on the task *Text Messaging* from Handout 2. The video for this activity shows the follow-up lesson in which pupils compare their own work with carefully selected samples of other pupils' work, also provided on the handout.

Before watching the video clip, familiarise yourself with the task, and with the sample work. Try to anticipate the issues that will arise as this work is discussed by pupils.



Now watch the pupils as they assess the sample work, and then go on to improve their own work.

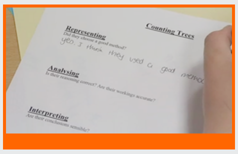
- What aspects of the provided work do pupils attend to?
- What criteria do pupils use as they assess the sample work?
- What are pupils learning from the sample work?

Teachers sometimes comment that some pupils attend more to the neatness of the sample work than to the quality and communication of the reasoning employed. Other teachers are concerned that pupils will uncritically copy sample work.

- How do you respond to these concerns?
- What criteria would you use for choosing sample work to use with pupils?

It is important that pupils spend time developing their own approaches before seeing sample work and that this work is chosen to suggest alternative representations and approaches that students have not previously considered. In addition it is helpful if the sample work illustrates common errors that will prompt discussion. If the sample work is chosen carefully in this way, and pupils are encouraged to be critical, then they will learn a great deal from discussing it.

Activity 3 **Discuss how pupils can assess their own work** **20 minutes**



Often when pupils have finished a piece of work, they want to move on. They don't want to re-examine it, polish it, or present it so that other people can understand and follow their reasoning.

In the video, both Emma and Shane ask their pupils to assess and improve each other's work. Emma uses the *Golden Rectangles* task and has collated a selection of her own pupils' work on this task into a poster. She has also prepared a simplified version of the progression steps to help her pupils analyse this work. The task and the original progression steps may be found on Handout 2. Note that pupils may be heard referring to a 'traffic lights' scheme that Emma uses in her Mathematics lessons: here, 'green', 'amber' and 'red' refer to decreasing levels of understanding.

Shane used the *Counting Trees* task and has prepared a less structured sheet to help his pupils assess each other's work. This sheet contains the names of the Key Processes.

Familiarise yourself with the tasks and then watch the video extracts of Shane's and Emma's lessons.

- What observations do pupils make about each other's work?
- How might this help them to improve their own work?
- Compare Emma's simplified progression steps with Shane's less structured sheet.
- Compare the use of work from within the pupils' own class to the use of the sample responses used in activity 2.

The progression steps may help pupils to develop an awareness of how Key Processes relate to particular tasks, and recognise how they can improve their responses. For the steps to be used in this way the language will need to be adapted to the class and specific 'answers' will need to be removed. The less structured approach already assumes that pupils have some understanding of what the Key Processes mean and that pupils can apply these to the task in hand.

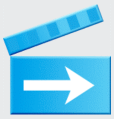
Teachers have commented that pupils are sometimes more able to be critical of sample responses that are taken from sources outside the classroom, when they cannot be identified. When giving feedback to members of their own class, personal relationships come into play. Classroom cultures may need to be developed where ideas and work may be criticised without individuals feeling threatened and exposed.

Activity 4 Plan to use peer and self-assessment strategies 10 minutes

Plan when you will allow pupils time to tackle one of the assessment problems, individually or in pairs, *without* your guidance.

Plan how and when you will revisit the task and allow pupils to assess other pupil's work - either work from their classmates or from the sample responses in Handout 2.

Make sure pupils have an opportunity to discuss the importance of the Key Processes, and sufficient time to revise their own work in the light of the comments.



If you have time, it may help your planning if you watch at least one of the videos showing more of Emma, Shane and Sheena's lessons.

When planning your lesson, you may find it helpful to watch the video lesson summaries for Shane, Emma and/or Sheena for ideas.

If you are working on this module with a group, it is helpful if each participant chooses the same assessment task, as this will facilitate the follow-up discussion.

This is the end of the *Introductory session*. After you have tried out your lesson with your own pupils, return for the *Follow-up session*. Resources to support the lessons, and suggested lesson plans, can be found in the *Into the classroom* session.

Reference:

Black, P., & Wiliam, D. (1998). *Inside the black box: raising standards through classroom assessment*. London: King's College London School of Education 1998.

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Into the classroom

Involving pupils in self- and peer-assessment

How can pupils help each other to use the Key Processes?

The following suggestions describe one possible approach to self- and peer-assessment. Pupils are given a chance to tackle a problem unaided, allowing you to assess their thinking and to identify pupils that need help. This is followed by formative lesson in which pupils collaborate, reflect on their work and try to improve it.

Before the lesson — 20 minutes

Before the lesson, perhaps at the end of a previous lesson, ask pupils to attempt one of the assessment tasks, *Text messages, Golden rectangles or Counting Trees,* on their own. Pupils will need calculators, pencils, rulers, and squared paper.

The aim is to see how able you are to tackle a problem without my help.

- *You will not be told which bits of maths to use*
- *There are many ways to tackle the problem - you choose*
- *There may be more than one 'right answer'*

Don't worry if you cannot understand or do everything because I am planning to teach a lesson on this next in the next few days.

Collect in pupils' work and review a sample of it. Look carefully at the range of methods pupils use and the quality of the reasoning. Try to identify particular pupils who have struggled and who may need support. Also look out for pupils that have been successful. These may need an extension activity to challenge them further.

Re-introduce the problem to the class — 5 minutes

Begin the lesson by briefly reintroducing the problem:

Do you remember the problem I asked you to have a go at last time?
Today we are going to work together and try to improve your first attempts.
Even if you got most of it right first time, you will learn something because there are different ways to tackle the problem.

At this point, choose between the Track A *or* Track B. Either decide to let pupils assess and improve their own work, *or* offer them the provided samples of work to assess. There won't be time for both!

Track A: Using pupils' own work

Track A: Pupils assess and improve their own work 15 minutes

Ask pupils to work in pairs or threes and give each group a large sheet of card and a felt-tipped pen. Give each group back their initial attempts at the problem.

> *I want you to look again at you answers, but this time work as a group.*
> *Take it in turns to describe your attempt to the rest of the group.*
> *After each suggestion, the others in the group should say what they like about your method and also where they think it can be improved.*
>
> *After you have all done this, I want you to work together to produce a better answer than you did separately.*
> *Make a poster showing your best ideas.*
> *It doesn't have to be beautiful, but it should show you thinking.*

Go round the room, listening, assessing their thinking and making appropriate interventions. Listen specifically to pupils that struggled with the task when they worked alone, and offer them support. If pupils have succeeded and their work is correct, provide one of the planned extensions.

Track A: Pupils exchange and comment on each others' work 15 minutes

Ask pupils to exchange their posters with another pair and issue each group with a copy of the "progression steps" framework for the task – one that is written in pupil-friendly language.

> *On a separate sheet of paper, write comments on:*
> - *Representing: Did they choose a good method?*
> - *Analysing: Is the reasoning correct – are the calculations accurate?*
> - *Interpreting: Are the conclusions sensible?*
> - *Communication: Was the reasoning easy to understand and follow?*

As they do this, go round encouraging pupils to read the work carefully and comment on the points mentioned. You may need to help them understand what the 'progression steps' mean. When pupils have commented on the work, one person from the group should take the poster to the group that produced it, and explain what needs to be done for the work to be improved.

Track A: Pupils improve their own work 5 minutes

Give groups a little time to absorb the comments and time to further improve their ideas.

Track A: Plenary discussion on approaches and changes 15 minutes

Towards the end of the lesson hold a discussion on the approaches used and the changes that have been made:

> *What changes have you made to your initial work?*
> *Why is it now better than it was before?*

Collect in the work and assess how the thinking has improved.

Track B: Using the provided sample work

Track B: Pupils assess provided sample work — 15 minutes

Give out the sample student work.

> *These samples of work were taken from another class. I want you to imagine that you are their teacher. This work may give you ideas you haven't thought of. It is also full of mistakes!*
>
> *I want you to comment on each of the following themes:*
> - *Representing: Did they choose a good method?*
> - *Analysing: Is the reasoning correct – are the calculations accurate?*
> - *Interpreting: Are the conclusions sensible?*
> - *Communication: Was the reasoning easy to understand and follow?*

In this way, pupils will become more aware of what is valued in their work – the Key Processes of representing, analysing, interpreting and communicating.

Listen to their discussions and encourage them to think more deeply. Encourage pupils to say what they like and dislike about each response and ask them to explain their reasons.

Track B: Pupils assess sample work using "progression steps" — 10 minutes

After pupils have had time to respond freely, issue each group with a copy of the "progression steps" framework for the task – one that is written in pupil-friendly language.

> *This framework may give you further ideas.*
> *Where would you put the work on the framework?*

Track B: Plenary discussion of the sample work — 15 minutes

Project each piece of sample student work on the board and ask pupils to comment on it:

> *What can we say about this piece of work?*
> *Share some of the comments you wrote.*
>
> *What did you think of the methods they chose?*
> *Which method did you like best? Why was this?*
>
> *Did you find any mistakes in their work?*
>
> *Do you agree with their conclusions?*

Track B: Working in pairs: Pupils improve their own work. — 10 minutes

Now using what they have learned, ask pupils to work together to improve their own solutions. As they do this, ask pupils to explain their thinking.

> *Max, tell me what you have done to improve your own solution.*

Collect examples of pupils' work for the follow-up session. Try to assess how much pupils have learned from the sharing session.

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Follow-up session

Involving pupils in self- and peer-assessment

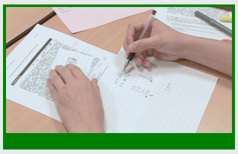
How can pupils help each other to use the Key Processes?

Activity 1 — Report and reflect on the assessment lesson — 20 minutes

Take it in turns to describe your experiences of using self- and peer-assessment.

- How did your pupils perform on the task, unaided?
- How did pupils assess the provided responses and the work of their peers?
  What aspects did they attend to?
- How did pupils make use of the 'progression steps'?
  Did these help pupils to understand the Key Processes?
- How well did pupils react to and use the evidence to improve their own work?
- What are the implications of this lesson for your future lessons?

Activity 2 — Discuss strategies for differentiation — 10 minutes



Reflect on your normal teaching practices. When you assess classes, you begin to realise the considerable individual differences in pupils and they have very different learning needs. Some pupils need more support, while others need a greater challenge.

- How do you *normally* deal with range of different learning needs of your pupils?
- Discuss the advantages and disadvantages of the four strategies shown on Handout 3
- Compare your views with the comments given on Handout 4

The strategies suggested on the handout are:

- *Differentiate by quantity?* When pupils appear successful, you provide them with a new problem to do
- *Differentiate by task?* You try to give each pupil a problem that is matched to their capability
- *Differentiate by outcome?* You use open problems that encourage a variety of possible outcomes
- *Differentiate by level of support?* You give all pupils the same problem, but then offer different levels of support, depending on the needs that become apparent

The first two of these approaches are unhelpful, particularly when developing Key Processes, for the reasons identified in Handout 4. Bowland tasks are 'open' in the sense that they encourage a variety of approaches. Their difficulty is not merely related to their apparent mathematical 'content', but is also related to the familiarity of the context, the complexity of information within the problem, the connections that need to be made, the length of the chains of reasoning required, and so on. Where one pupil chooses algebra, another may choose a numerical approach and the demands of each method will be different.

Activity 3 Discuss ways of helping pupils that struggle 10 minutes

As well as finding the tasks challenging, pupils may find the whole idea of self and peer assessment difficult. They are being asked to reflect on the methods and processes that they and others have used. Think again about your lessons using the Bowland assessment tasks.

- How might you help those who struggle with the task?
- How can you help those who struggle with the whole idea of peer assessment?

Teachers have found that when pupils get stuck with a task, then they may be considerably helped by:

- discussing their difficulty with a partner (not necessarily their neighbour);
- looking at some samples of other pupils' attempts (however rough) - these will suggests new ways to access and approach the task.

As soon as the teacher gives detailed guidance on what to do, the pupils are unable to make strategic decisions for themselves. Such guidance should therefore only be given as a last resort, after pupils have been allowed to struggle and help each other.

We have found that most pupils enjoy and value self and peer assessment. Some however, may be unused to revisiting tasks and reflecting on earlier work and may not therefore appreciate the value of discussing different solution methods in depth. "When I know the answer, what point is there in discussing the problem further and looking at other people's work?" Such pupils prefer to 'get on' and tackle new tasks. We have found that it is important to carefully explain the purpose of peer assessment to pupils meeting it for the first time.

Activity 4 Discuss ways of stretching pupils that succeed 10 minutes

Some pupils may have done very well at the problems, even at the very beginning. Others may have worked well and finished quickly. It is a good idea to plan for such eventualities.

Think back to your own lesson:

- When pupils succeeded, how did you extend their thinking?
- What alternative approaches to the task did you, or could you suggest?
- What extensions to the task did you, or can you suggest?

If you wish, you can watch a video of Shane, Emma and Sheena discussing this issue.

Even if pupils succeed in the problems, they can still learn a great deal by revisiting them. Emma, Sheen and Shane suggest that pupils may be encouraged to:

- find alternative or more elegant ways of representing and tackling the task;
- make up their own variants or extensions to tasks
- devise their own "progression steps", to develop their understanding of Key Processes.

You may be like to suggest your own possible extensions to the tasks. For example:

- *Text Messaging*: How long would it take to spread a piece of news around the school if each person sends a text message to four other people?
- *Counting Trees*: What method would you use if you were asked to estimate the number of beans in a jar?
- *Golden rectangles*: Suppose the adventurers were only given three stakes each? (and rename the task: Golden triangles).

Activity 5 Plan assessment strategies for future lessons 10 minutes

- How might you apply what you have learned about assessment to your other mathematics lessons?

- How might you embed peer assessment strategies in your scheme of work?

You may like to watch the video showing the teachers discussing the wider implications of assessing Key Processes.

Further Reading

See Handout 5 for suggested further reading.

1 Strategies for helping pupils to become more aware of Key Processes

1. Using a poster or handout

Make a poster showing the generic list of Key Processes and display this on the classroom wall. Refer to this habitually, while pupils work on unstructured problems, so that they become more aware that your goals for the lesson are for them to become more able to *represent, reason, interpret, evaluate* and *communicate*.

2. Creating task-specific hints

Before the lesson, prepare some task-specific hints that apply the generic processes to the particular problem in hand. When pupils are stuck, give them the appropriate hint either on paper or orally. For example, you could ask: *"Can you use a table or graph to organise this data?"; "What is fixed and what can you change in this problem?"; "What patterns can you see in this data?".*

3. Asking pupils to assess provided samples of work

After students have worked on a task, present them with some prepared, sample responses from other students. These solutions provide alternative strategies students may not have considered and may also contain errors. Ask pupils to pretend they are examiners. The students rank order these solutions, along with their own response, giving explanations as to why they think one response is better than another.

4. Using prepared 'progression steps'

Students evaluate sample responses as in (3) above, but this time you also provide them with *prepared progression steps* that highlight the key processes. Students use these to evaluate the work. End the lesson by sharing what has been learned from this process.

5. Asking pupils to assess each other's work.

After tackling a Bowland task in pairs, pupils exchange their work. Each pair of pupils is given the work of another pair. Pupils make suggestions for ways of improving each solution and stick these on the work using "sticky" notes. These comments are passed back to the originators, who must then produce a final, improved version based on the comments received. This is a more challenging strategy for the teacher than (3), as the issues that arise will be less predictable.

6. Pupils interview each other about the processes they have used.

When pupils have finished working on a task, ask them to get into pairs. Each member of a pair interviews his or her partner about their approach and the processes they have used while working on the task. The teacher may provide some pre-prepared questions to assist in this. After noting down the replies, pupils change roles. Suitable questions might be:

- *What approach did you take?*
- *Which processes did you use (from a provided list)?*
- *How could this work be improved?*
- *What could you have done differently?*
- *Is there still something you are confused by?*

2 Three assessment tasks and five sample responses on each

Text Messaging

1. How many text messages are sent if four people all send messages to each other?
2. How many text messages are sent with different numbers of people?
3. Approximately how many text messages would travel in cyberspace if everyone in your school took part?
4. Can you think of other situations that would give rise to the same mathematical relationship?

This was adapted from Sending texts *– a task from the Nuffield Foundation's* Applying Mathematical Processes *project – see http://www.nuffieldcurriculumcentre.org/*

Follow-up task for students

Look carefully at the following extracts of work from other students. Imagine you are their teacher. Go through each piece of work and write comments on each one.

- Have they chosen a sensible method?
- Are the calculations correct?
- Are the conclusions sensible?
- Is the work easy to understand?

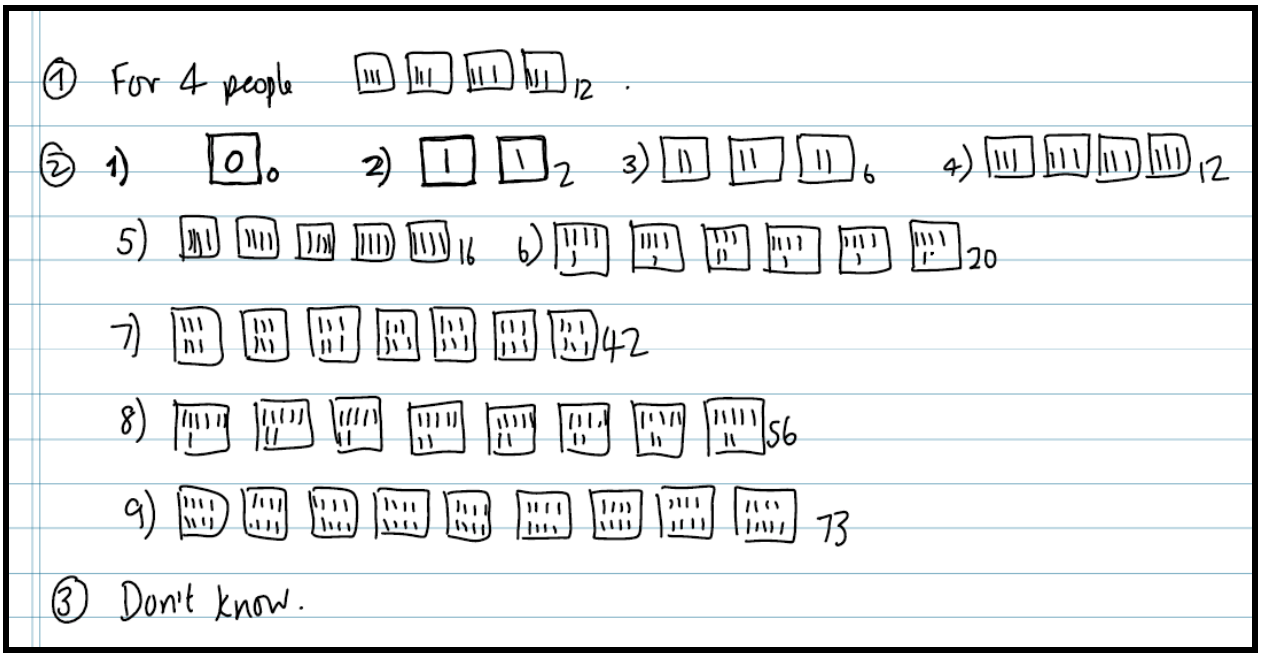
Name	**Comments**
Tom	
Sam	
Chris	
Lily	
Marvin	

Now try to write out an answer that is better than all of them!

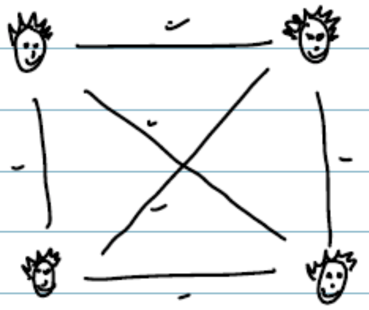
Tom's answer

Celia Send's one to Tracey =1
Tracey Send's one to Celia =1
Tracey Send's one to Maria =1
Maria Sends one to anne-maria =1
Anne-marie Send's one to Eelia =1
Celia Send's one to anne-Marie =1
Maria Send's one to Tracey =1
Tracey send's one to Annemarie =1
Maria Send's one to Celia =1

Sam's answer



Chris's answer



= 6 texts

3 

People	1	2	3	4	5
texts	0	1	3	6	10

Lily's answer

	Amy	Belinda	Suzie	Mary	Tom
Amy	—	Text	Text	Text	Text
Belinda	Text	—	Text	Text	Text
Suzie	Text	Text	—	Text	Text
Mary	Text	Text	Text	—	Text
Tom	Text	Text	Text	Text	—

= 12 texts for 4 people

Tom adds 8 more texts = 20 altogether.

For more people you add extra rows and colums.

Marvin's answer

$4 \times 3 = 12$ So there are 12 messages with 4 people.

With eight people there will be $8 \times 7 = 56$ messages

With a thousand people there will be $1000 \times 999 = 999000$ messages

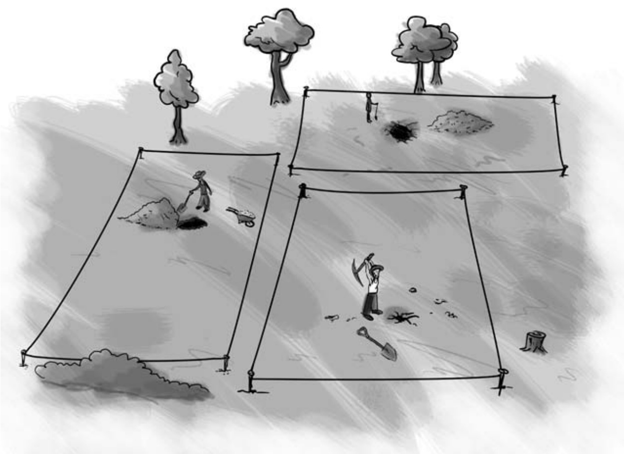
The formula is number of people x one less than this because
you dont send a text to yourself.

Progression in key processes

	Representing	**Analysing**	**Interpreting and evaluating**	**Communicating**
	Represents some individual text messages that are sent.	Works out the number of text messages for four people correctly.	Says that everyone sends the same number of messages.	Shows how the answer was found.
PROGRESSION	Uses marks or diagrams to show the texts. Chooses to use repeated addition.	Increases the number of people in an organised way. Correctly works out the number of texts sent to different numbers of people.		Shows the method clearly and where the answers come from.
	Chooses to use multiplication to work out the number of texts sent.	Finds a correct pattern in the results. The reasoning is based on particular examples.	Explains the result for a number of people other than 4. Finds a correct rule for calculating the number of texts.	Explains how the rule links to the context of sending texts.
	Chooses to use algebra to show the general case.	The reasoning moves from looking at particular examples to more general cases.	Makes and justifies correct general statements relating the number of texts to the number of people. Makes and justifies statements for large numbers of students.	Writes a complete and concise summary with clear links to the original context. Discusses mathematical similarities and differences between sending texts and other contexts – e.g. matches in a football league.

Golden Rectangles

In the 19th century, many adventurers travelled to North America to search for gold.
A man named Dan Jackson owned some land where gold had been found.
Instead of digging for the gold himself, he rented plots of land to the adventurers.



Dan gave each adventurer four wooden stakes and a rope measuring exactly 100 metres.

Each adventurer had to use the stakes and the rope to mark off a rectangular plot of land.

1. Assuming each adventurer would like to have the biggest plot, how should he place his stakes?
   Explain your answer.

Read the following proposition:

“Tie the ropes together! You can get more land if you work together than if you work separately.”

2. Investigate whether the proposition is true for two adventurers working together, still using four stakes.

3. Is the proposition true for more than two people?
   Explain your answer.

Follow-up task for students

Look carefully at the following extracts of work from other students. Imagine you are their teacher. Go through each piece of work and write comments on each one.

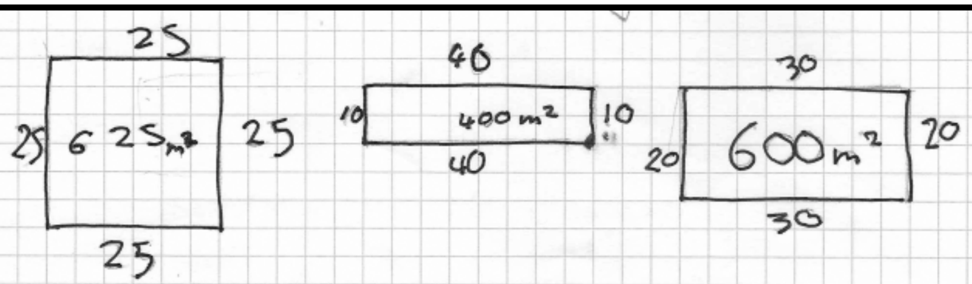
- Have they chosen a sensible method?
- Are the calculations correct?
- Are the conclusions sensible?
- Is the work easy to understand?

Name	Comments
Alvin	
Bernie	
Chris	
Danny	
Elsie	

Now try to write out an answer that is better than all of them!

Alvin's answer

1

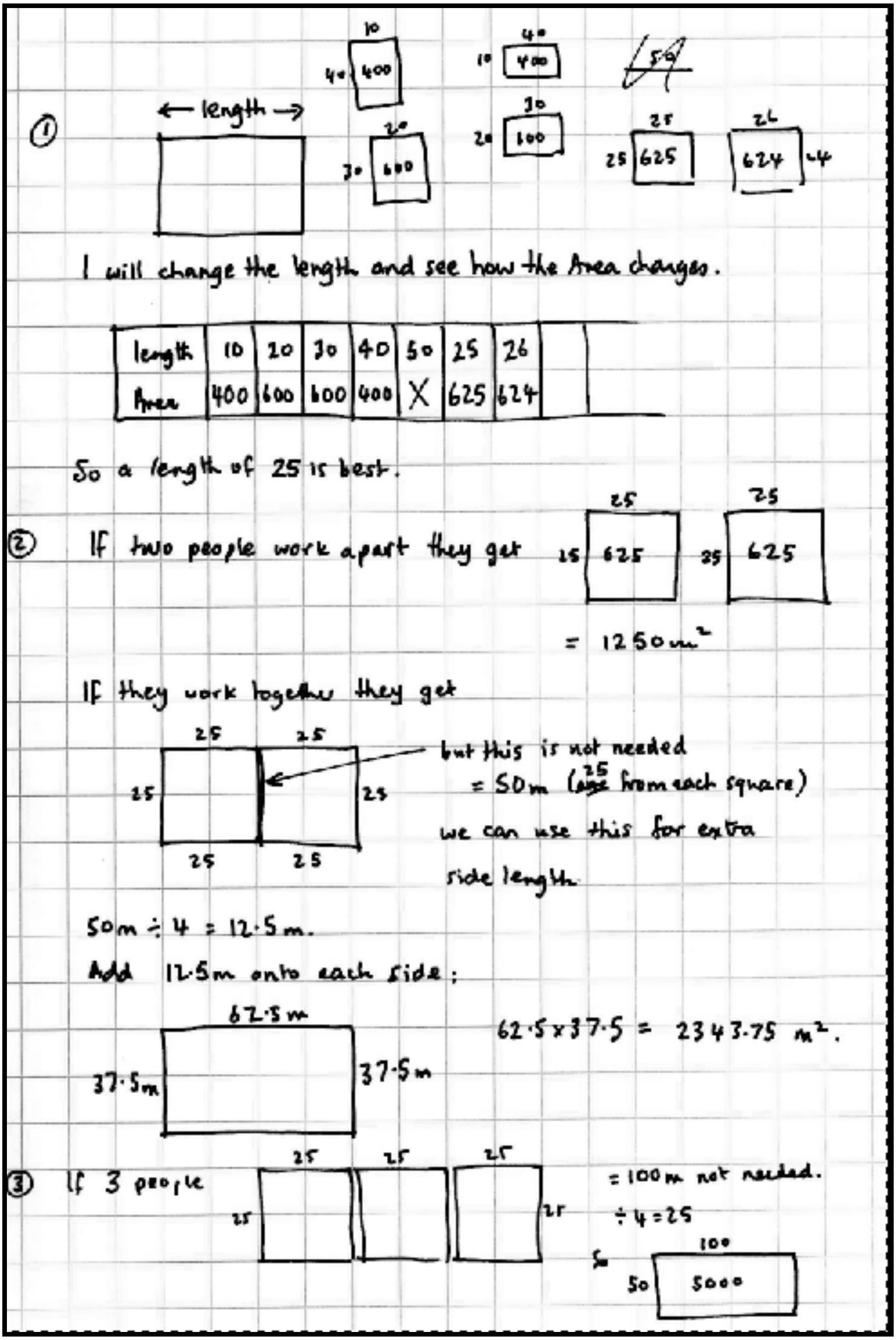


If you want the biggest plot,
I think you need the biggest area,
so what I did was draw the rectangles
out and I found out that the
more equal it is the bigger the area.

2 It is better to work on your
own because if you work together
there will be a bigger area but
you will have to half it with
the other person, For example, If
you combine the ropes you will have
200m, If you do 50 × 50 to find
the area it will be 2500m² but you
will need to half that with other person
so that will give you 1250m², so
you will have more to do. So
it is easier to work on your own.

3 No it is not true for more
than 2 people, they will have to
work harder.

Bernie's answer



① ← length →

I will change the length and see how the Area changes.

length	10	20	30	40	50	25	26		
Area	400	600	600	400	X	625	624		

So a length of 25 is best.

② If two people work apart they get 25 625 25 625

= 1250m²

If they work together they get

but this is not needed = 50m (25 from each square)

we can use this for extra side length.

50m ÷ 4 = 12·5m.

Add 12·5m onto each side:

62·5m 37·5m 37·5m

62·5 × 37·5 = 2343.75 m².

③ If 3 people 25 25 25 25 25

= 100m not needed.

÷ 4 = 25

So 100 50 5000

Chris's answer

a $25 \times 25 = 625\ m^2$

$30 \times 20 = 600\ m^2$

$40 \times 10 = 400\ m^2$

He should place the stakes in a rectangulae because then he has the most land. But the rectangle need tobe 30 x 20m.

b With two ropes of 100 m, You can get a bigger amaunt of land. If you take 55m x 45m, you get more than the dubble amaunt of land. 55x45=2475.

$2475\ m^2 : 2 = 1237.5\ m^2$

c Yes, because you can make the plot of land bigger in that way everyone has more land.

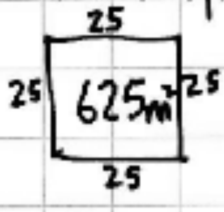
If the plot of land is 80 x 70, the land is 5600 m².

$5600\ m^2 : 3 = 1866.67 \rightarrow 1866.7\ m^2$ per person.

That is more land.

Danny's answer

① He should place his stakes in a square to give the biggest area like this



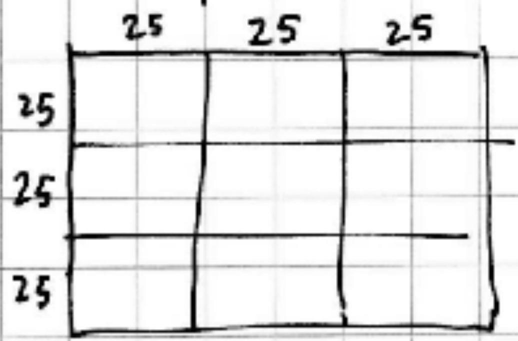
② If two adventurers work together they will have $200m^2$ of rope so they can make a square twice as long and wide.

= 4x area.

This is much better than 2x area.

③ If three work together they will have $300m^2$ of rope so they can make a square three times as long and wide

= 9x area



This is much better than 3x area.

I think that the area goes up by square numbers each time.

Elsie's answer

a: 4 x 25 metres → area = 25 x 25 = $\boxed{625\ m^2}$
2 x 20 & 2 x 30 → area = 20 x 30 = 600 m^2
2 x 10 & 2 x 40 → area = 10 x 40 = 400 m^2

So 4 x 25 metres would make the biggest area

b 2 x 100 metres of Rope = 200 m.
4 x 50 metres → area = 50 x 50 = $\boxed{2500\ m^2}$
2 x 20 & 2 x 80 → area = 20 x 80 = 1600 m^2
2 x 30 & 2 x 70 → area = 30 x 70 = 2100 m^2
2 x 40 & 2 x 60 → area = 40 x 60 = 2400 m^2
2 x 10 & 2 x 90 → area = 10 x 90 = 900 m^2

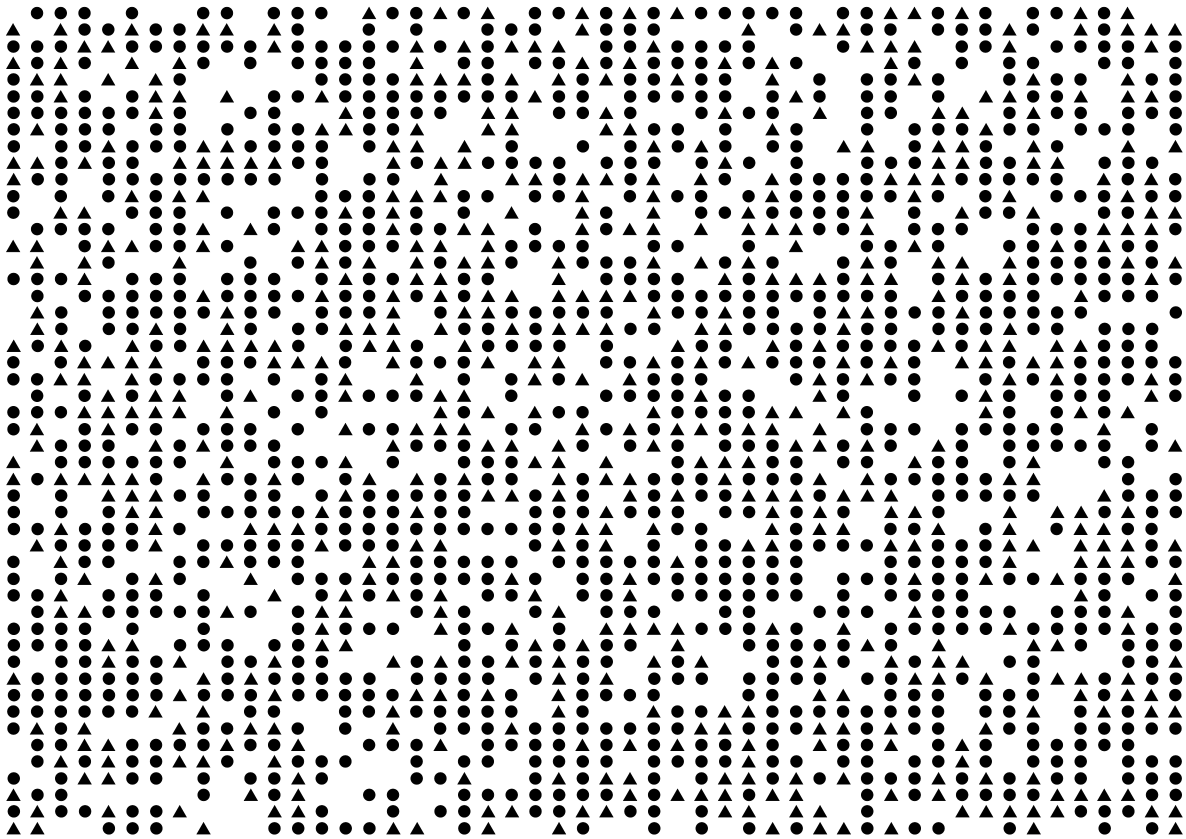
So the proposition is true, working together will deliver much more land to dig for gold.

c for example: 300 metres of ~~8~~ Rope
4 x 75 metres → area = 75 · 75 = 5625 m^2
So how longer the rope is, how bigger the land will be.
400 metres of rope (4 people working together)
4 x 100 metres → area = 100 · 100 = 10000 m^2

Progression in key processes

	Representing	Analysing	Interpreting and evaluating	Communicating
PROGRESSION	The student draws one or two rectangles with a perimeter of 100m.	The student works out the areas of their rectangles correctly.	The student draws several rectangles but not a square and the justification is incorrect or omitted.	The work is communicated adequately, but there are gaps and/or omissions.
	Draws several rectangles.	Calculates the areas of their rectangles and attempts to come to some generalisation.	Realises that different shapes have different areas but comes to incorrect or incomplete conclusion.	The work is communicated clearly and the reasoning may be followed.
	Draws several, correct rectangles for an adventurer working alone and for 2 working together. May draw far too many rectangles.	Calculates the areas correctly and finds that a square is best for 1 adventurer and that 2 working together do better than alone.	Attempts to give some explanation for their findings.	The work is communicated clearly and the reasoning may be easily followed.
	Draws an appropriate number of rectangles and collects the data in an organised way.	Calculates the correct areas, finds that a square is best for 1 adventurer and that 2 working together do better than alone. Finds a rule or pattern in their results.	Gives reasoned explanations for their findings.	Explains work clearly and may consider other shapes.

Counting Trees



This diagram shows some trees in a plantation.

The circles ● show old trees and the triangles ▲ show young trees.
Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

1. What method could he use to estimate the number of trees of each type?
   Explain your method fully.

2. On your worksheet, use your method to estimate the number of:
   (a) Old trees
   (b) Young trees

Follow-up task for students

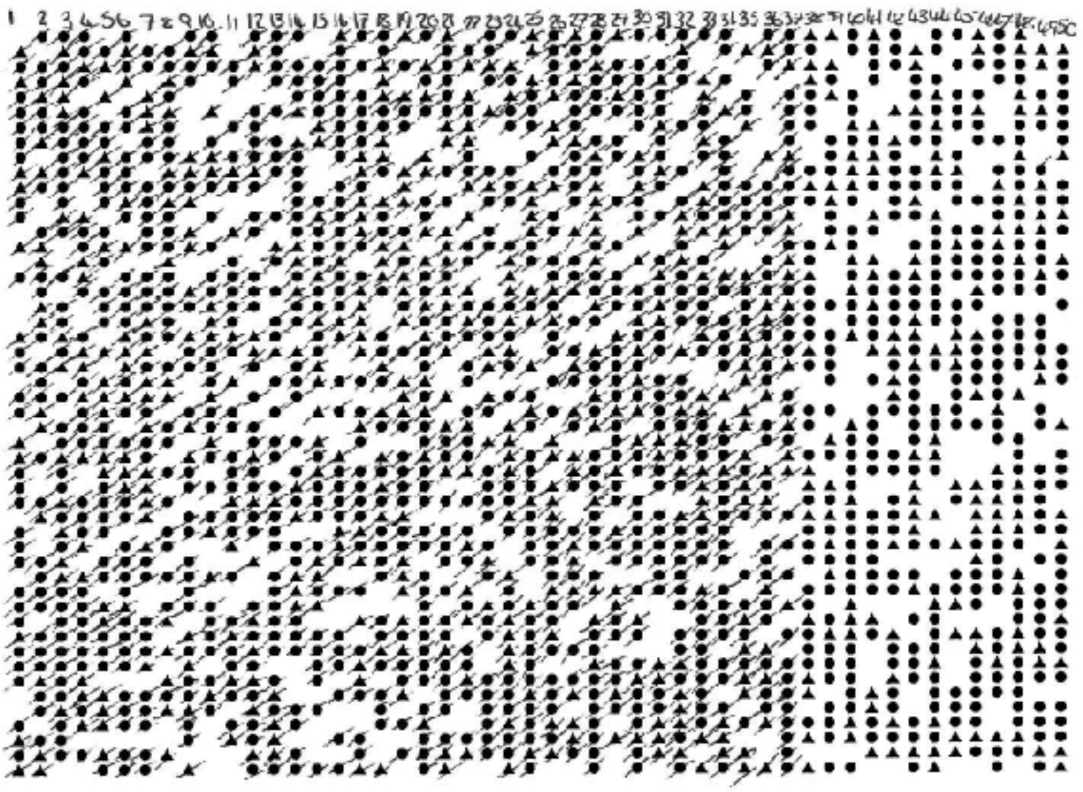
Look carefully at the following extracts of work from other students. Imagine you are their teacher. Go through each piece of work and write comments on each one.

- Have they chosen a sensible method?
- Are the calculations correct?
- Are the conclusions sensible?
- Is the work easy to understand?

Name	Comments
Sarah	
Laura	
Jenny	
Woody	
Amber	

Now try to write out an answer that is better than all of them!

Sample response: Sarah



1-	▲=11	●=27	2-	▲=13	●=25	3-	▲=13	●=31
4-	▲=14	●=26	5-	▲=16	●=23	6-	▲=12	●=30
7-	▲=14	●=27	8-	▲=12	●=26	9-	▲=14	●=21
10-	▲=12	●=22	11-	▲=9	●=24	12-	▲=13	●=26
13-	▲=7	●=30	14-	▲=11	●=28	15-	▲=13	●=24
16-	▲=7	●=30	17-	▲=17	●=22	18-	▲=16	●=25
19-	▲=19	●=19	20-	▲=12	●=30	21-	▲=15	●=24
22-	▲=15	●=21	23-	▲=9	●=25	24-	▲=19	●=22
25-	▲=15	●=25	26-	▲=15	●=25	27-	▲=15	●=23
28-	▲=15	●=30	29-	▲=10	●=26	30-	▲=12	●=29
31-	▲=13	●=26	32-	▲=14	●=27	33-	▲=17	●=25
34-	▲=10	●=31	35-	▲=17	●=17	36-	▲=13	●=25
37-	▲=12	●=29	38-	▲=	●=	39-	▲=	●=
40-	▲=	●=	41-	▲=	●=	42-	▲=	●=
43-	▲=	●=	44-	▲=	●=	45-	▲=	●=
46-	▲=	●=	47-	▲=	●=	48-	▲=	●=
49-	▲=	●=	50	▲=	●=			

estimate- ▲=670 ●=1320

▲ = 11+13+13+14+16 ×10 ● = 27+25+31+26+23 ×10

Sample response: Laura

① You could multiply the number of trees in the length by the number of trees in the width and then half your answer.

② a. Old trees -644
Young trees - 644

width - 33
length - 39.

33x39= 1287
1287÷2= 643.5-644

Sample response: Jenny

1. there are 38 trees in each column
there are around 11 young trees
and around 27 old ones
33 trees in each row so

11 x33 = 363
27 x33 = 891
1254

2.
a. 11x33 = 363 = new trees.
b. 27 x 33 = 891 = old trees.

Sample response: Woody

2 columns has 21 young trees
55 old

50 columns is approx
50 ÷ 2 = 25
25 × 21 = amount of young trees = 525
25 × 55 = amount of old trees = 1,375
rounded up
young 530
old 1,380

Sample response: Amber

Counting trees

1. If Tom draws a 10X10 square round some trees and counts how many old and new there are. There are 50 rows and 50 columns altogether so he must multiply by 25. He could do this a few times to check and then take the average.

2.

53 old	x 25	=	1325	old
28 new	x 25	=	700	new
19 spaces	x 25	=	475	spaces
100			2500	

1325 + 1200 ÷ 2 = 1262·5
700 + 875 ÷ 2 = 787·5

Check

48 old	x 25	=	1200	old
35 new	x 25	=	875	new
17 spaes	x 25	=	425	spaes
100			2500	

So about 1263 old trees and 788 new Trees

Progression in key processes

	Representing	Analysing	Interpreting and evaluating	Communicating and reflecting
	Chooses a method, but this may not involve sampling. E.g. Counts all trees or multiplies the number of trees in a row by the number in a column.	Follows chosen method, possibly making errors. E.g. Does not account for different numbers of old and young trees or that there are gaps.	Estimates number of new and old trees, but answer given is unreasonable due to method and errors.	Communicates work adequately but with omissions.
PROGRESSION	Chooses a sampling method but this is unrepresentative or too small. E.g. tries to count the trees in first row and multiplies by the number of rows.	Follows chosen method, mostly accurately. E.g. May not account for different numbers of old and young trees or that there are gaps.	Estimates number of new and old trees, but answer given is unreasonable due mainly to the method.	Communicates reasoning and results adequately, but with omissions.
	Chooses a reasonable sampling method.	Follows chosen method, mostly accurately.	Estimates a reasonable number of old and new trees in the plantation. The reasonableness of the estimate is not checked. E.g. by repeating with a different sample.	Explains what they are doing but explanation may lack detail.
	Chooses an appropriate sampling technique.	Follows chosen method accurately. Uses a proportional argument correctly.	Deduces a reasonable number of old and new trees in the plantation. There is some evidence of checking the estimate. E.g. Considers a different sampling method.	Communicates reasoning clearly and fully.

3 Meeting the needs of all pupils

Assessment reveals that all pupils have different learning needs.
How do you respond to this in your normal lessons?

Discuss and note down the advantages and disadvantages of each approach. Add your own ideas underneath.

Differentiate by quantity? When pupils appear successful, you provide them with a new problem to do. ....
Differentiate by task? You try to give each pupil a problem that is matched to their capability. ....
Differentiate by outcome? You use open problems that encourage a variety of possible outcomes. ....
Differentiate by level of support? You give all pupils the same problem, but then offer different levels of support, depending on the needs that become apparent. ....

4 Meeting the needs of all pupils – some comments to consider

Differentiate by quantity?
When pupils appear successful, you provide them with a new problem to do.

This approach is common, but it leads to pupils viewing the curriulum as a list of problems to do, rather than processes to acquire. This approach will not promote reflection on alternative methods for doing a problem - different ways of representing, analysing, interpreting and communicating.

Differentiate by task?
You try to give each pupil a problem that is matched to their capabilities.

But how does one know if a problem is suitable? We can only match a problem to a pupil if we have a profound understanding of both. Our view of the problem is usually based on our own way of doing it – and there may be many other approaches. We also have an imperfect and often prejudiced view of pupils' capabilities. We so easily judge pupils' 'mathematical ability' by their ability to carry out routine procedures they have recently been taught. Problem solving requires a different set of skills and may result in different pupils performing well. This approach also creates management difficulties as different problems are used with different pupils. This reduces possibilities for whole class discussions and sharing knowledge.

Differentiate by outcome?
You use more open problems that encourage a variety of possible approaches and outcomes.

This approach requires problems and situations that allow for such a variety to emerge. The Bowland problems are like this, but they do make considerable demands on pupils who are unfamiliar to problem solving. Many teachers comment that as soon as pupils begin to struggle, they want to 'leap in', 'take over' and structure the problem, so that pupils have clear steps to follow. This tendency undermines the very purpose of the lesson – to develop pupils' ability to use Key Processes in an autonomous way. On the other hand, too little guidance may result in prolonged failure and frustration. Some teachers therefore make it a rule that pupils should always help and share ideas with each other, before asking for help from the teacher.

Differentiate by level of support?
You give all pupils the same problem, but then offer different levels of support, depending on the needs that become apparent.

This approach avoids many of the difficulties described above. The support may be by other pupils, or by the teacher - orally, or in written form. In the lessons we have suggested, the teacher asks the pupils to attempt what they can unaided, then they are offered the support of their peers as ideas and approaches are shared and discussed. If further support is needed, then the teacher may supply this through questions that cause pupils to attend to particular features of the problem, or through more specific hints. Timing such help is critical. One of the important goals of problem solving is to allow pupils the experience of *struggling* with a problem for some time and experiencing the sense of achievement that arrives when the problem has been overcome. If we help pupils too quickly, we rob them of this experience.

5 Suggestions for further reading

Black, P., & Wiliam, D. (1998). *Inside the black box: raising standards through classroom assessment*. King's College London School of Education.
Now published by GL Assessment: http://shop.gl-assessment.co.uk

This short booklet offers a summary of the extensive research literature into formative assessment. It shows that there is clear evidence that improving formative assessment raises standards, and offers evidence showing how formative assessment may be improved. This booklet is essential reading for all teachers.

Black, P., & Harrison, C. (2002). *Working inside the black box: Assessment for learning in the classroom*. King's College London School of Education.
Now published by GL Assessment: http://shop.gl-assessment.co.uk

In this booklet, the authors describe a project with teachers in which they studied practical ways of implementing formative assessment strategies and the effect this had on learning. The section on peer-assessment and self-assessment (pages 10-12) are particularly relevant to this module.

Black, P., Harrison, C., Lee, C., Marshall, B., & Wiliam, D. (2003). *Assessment for learning: Putting it into practice*. Buckingham: Open University Press.

This book gives a fuller account of the earlier booklets *Inside the black box* and *Working inside the black box*. It discusses four types of action: questioning, feedback by marking, peer- and self assessment and the formative use of summative tests. The section on peer and self-assessment (pp 49-53) is particularly relevant to this module.

Hodgen, J., & Wiliam, D. (2006). *Mathematics inside the black box*. King's College London School of Education. Now published by GL Assessment: http://shop.gl-assessment.co.uk

This booklet applies the above findings specifically to Mathematics. It considers some principles for Mathematics learning, choice of activities that promote challenge and dialogue, questioning and listening, peer discussion, feedback and marking, and self and peer assessment. This booklet is essential reading for all mathematics teachers. Pages 9-10 are particularly relevant to this module.

