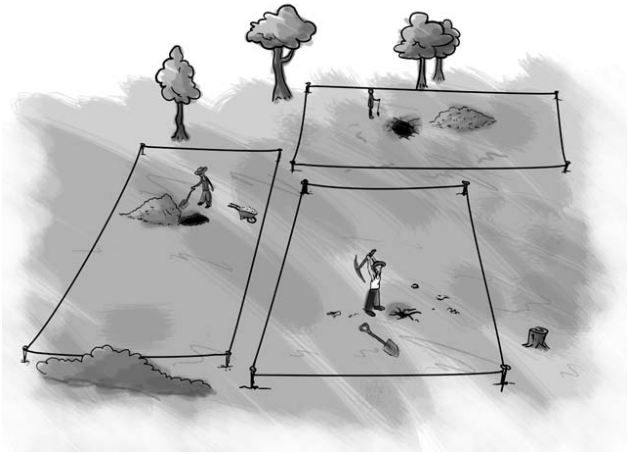
Golden Rectangles

In the 19th century, many adventurers travelled to North America to search for gold.
A man named Dan Jackson owned some land where gold had been found.
Instead of digging for the gold himself, he rented plots of land to the adventurers.



Dan gave each adventurer four wooden stakes and a rope measuring exactly 100 metres.

Each adventurer had to use the stakes and the rope to mark off a rectangular plot of land.

1. Assuming each adventurer would like to have the biggest plot, how should he place his stakes?
   Explain your answer.

Read the following proposition:

“Tie the ropes together! You can get more land if you work together than if you work separately.”

2. Investigate whether the proposition is true for two adventurers working together, still using four stakes.

3. Is the proposition true for more than two people?
   Explain your answer.

Follow-up task for students

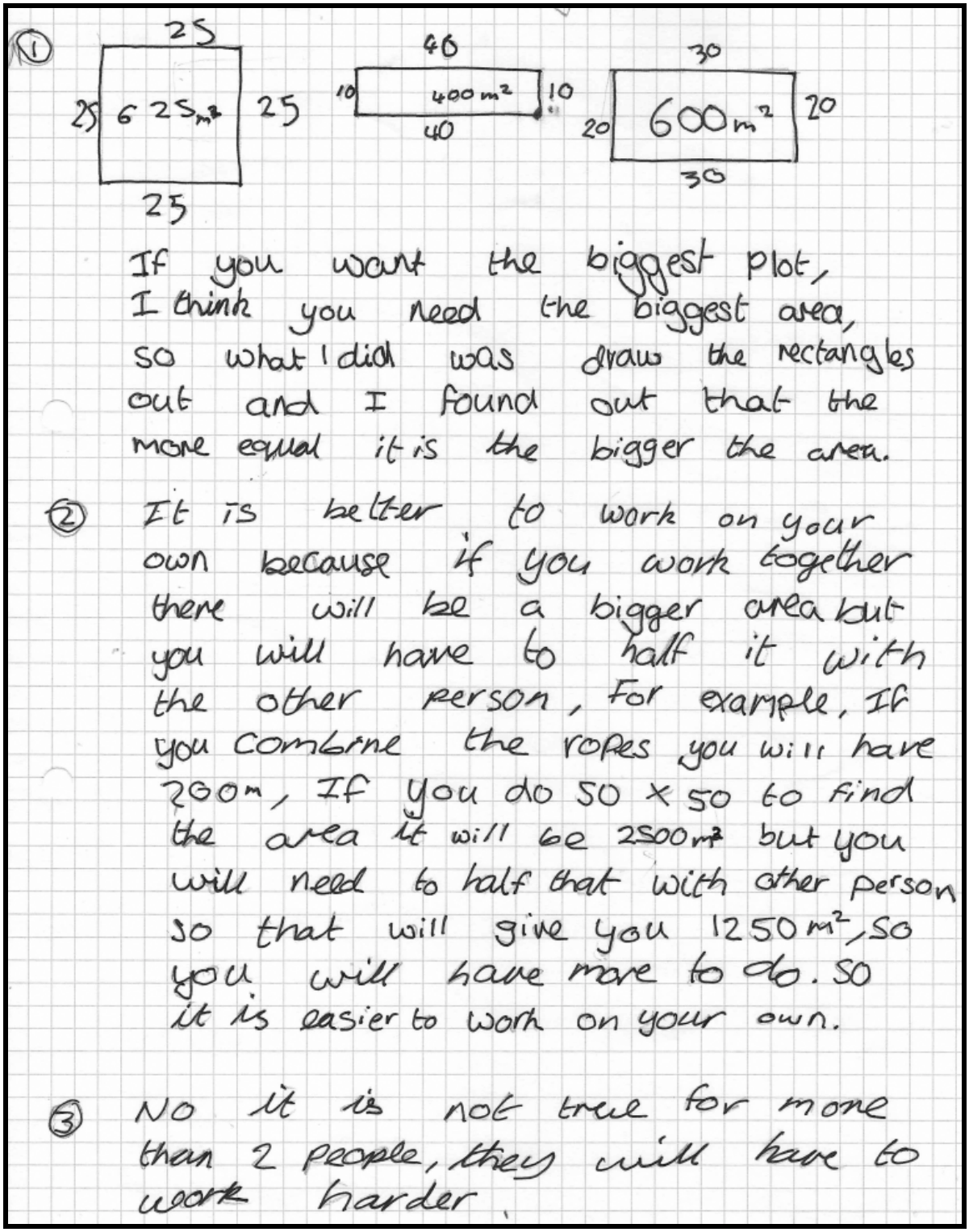
Look carefully at the following extracts of work from other students. Imagine you are their teacher. Go through each piece of work and write comments on each one.

- Have they chosen a sensible method?
- Are the calculations correct?
- Are the conclusions sensible?
- Is the work easy to understand?

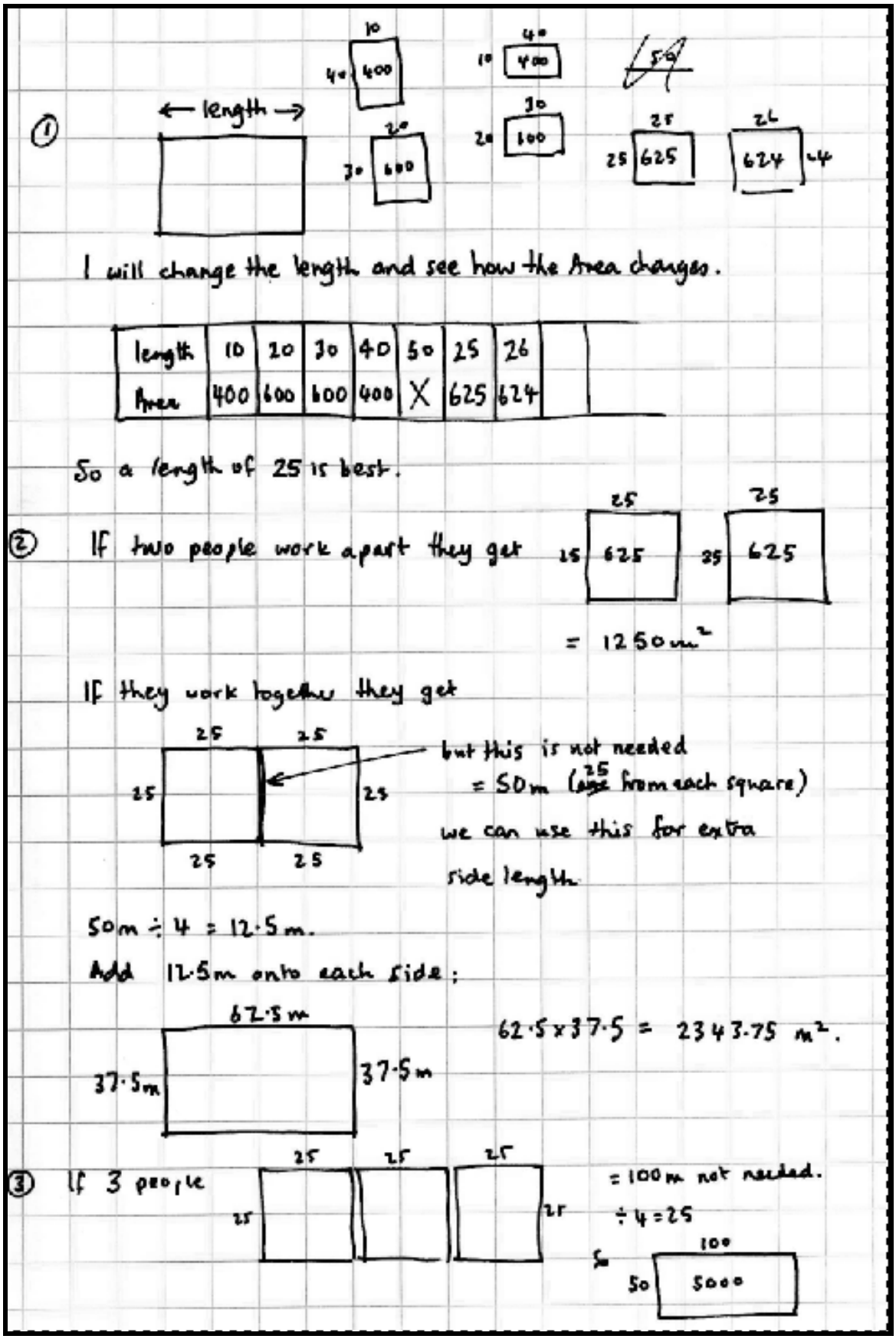
Name	Comments
Alvin	
Bernie	
Chris	
Danny	
Elsie	

Now try to write out an answer that is better than all of them!

Alvin's answer



Bernie's answer



Chris's answer

a $25 \times 25 = 625\ m^2$

$30 \times 20 = 600\ m^2$

$40 \times 10 = 400\ m^2$

He should place the stakes in a rechangulae because then he has the most land. But the rectangle need to be 30×20m.

b With two ropes of 100 m, you can get a bigger amount of land. If you take 55m x 45m, you get more than the dubble amount of land. $55 \times 45 = 2475$,

$2475\ m^2 : 2 = 1237.5\ m^2$

c Yes, because you can make the plot of land bigger in that way everyone has more land.

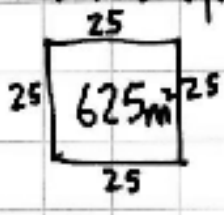
If the plot of land is 80×70, the land is $5600\ m^2$.

$5600\ m^2 : 3 = 1866.67 \rightarrow 1866.7\ m^2$ per person.

That is more land.

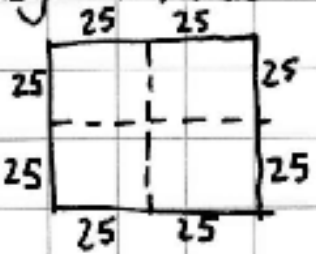
Danny's answer

① He should place his stakes in a square to give the biggest area like this



② If two adventurers work together they will have 200m² of rope so they can make a square twice as long and wide.

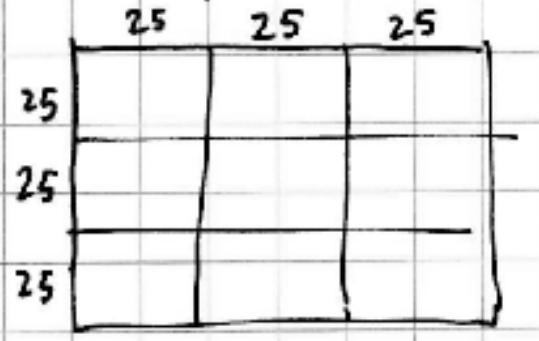
= 4x area.



This is much better than 2x area.

③ If three work together they will have 300m² of rope so they can make a square three times as long and wide

= 9x area



This is much better than 3x area.

I think that the area goes up by square numbers each time.

Elsie's answer

a: 4 x 25 metres → area = 25 x 25 = 625 m²
2 x 20 & 2 x 30 → area = 20 x 30 = 600 m²
2 x 10 & 2 x 40 → area = 10 x 40 = 400 m²

So 4 x 25 metres would make the biggest area

b 2 x 100 metres of rope = 200 m.
4 x 50 metres → area = 50 x 50 = 2500 m²
2 x 20 & 2 x 80 → area = 20 x 80 = 1600 m²
2 x 30 & 2 x 70 → area = 30 x 70 = 2100 m²
2 x 40 & 2 x 60 → area = 40 x 60 = 2400 m²
2 x 10 & 2 x 90 → area = 10 x 90 = 900 m²

So the proposition is true, working together will deliver much more land to dig for gold.

c for example: 300 metres of rope
4 x 75 metres → area = 75 · 75 = 5625 m²
So how longer the rope is, how bigger the land will be.
400 metres of rope (4 people working together)
4 x 100 metres → area = 100 · 100 = 10000 m²

Progression in key processes

	Representing	Analysing	Interpreting and evaluating	Communicating
PROGRESSION	The student draws one or two rectangles with a perimeter of 100m.	The student works out the areas of their rectangles correctly.	The student draws several rectangles but not a square and the justification is incorrect or omitted.	The work is communicated adequately, but there are gaps and/or omissions.
	Draws several rectangles.	Calculates the areas of their rectangles and attempts to come to some generalisation.	Realises that different shapes have different areas but comes to incorrect or incomplete conclusion.	The work is communicated clearly and the reasoning may be followed.
	Draws several, correct rectangles for an adventurer working alone and for 2 working together. May draw far too many rectangles.	Calculates the areas correctly and finds that a square is best for 1 adventurer and that 2 working together do better than alone.	Attempts to give some explanation for their findings.	The work is communicated clearly and the reasoning may be easily followed.
	Draws an appropriate number of rectangles and collects the data in an organised way.	Calculates the correct areas, finds that a square is best for 1 adventurer and that 2 working together do better than alone. Finds a rule or pattern in their results.	Gives reasoned explanations for their findings.	Explains work clearly and may consider other shapes.